Robust Controller Design for Formation Flight of Quad-Rotor Helicopters

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***Abstract*— In this paper we present an application of a recently developed strategy for robust distributed controller design for formations and show a way of including performance requirements in the design. The proposed synthesis method guarantees stability for all possible formations and arbitrary fast changes in the communication topology. The number of agents in the formation can also be chosen arbitrarily. We illustrate the results by performing a simulation of a formation flight of quad-rotor helicopters.**

I. INTRODUCTION

Advances in the field of computation and communication technologies as well as the ongoing process of miniaturization of embedded systems have sped up the development of multi-agent systems. It is characteristic for multi-agent systems that they are capable of communicating with each other to accomplish a common goal. Furthermore they are dynamically decoupled from each other and can interact autonomously with their environment. It seems to be natural that multi-agent systems can perform tasks which are beyond the capabilities of one agent. Besides unmanned aerial vehicles (UAVs) that are considered in this paper, application areas include autonomous underwater vehicles [1], [2], mobile robotics [3], automated highway systems [4] and satellite formation flying [5].

Although distributed systems suffer from latency and depend on the structure of the communication topology, thus requiring a more complex controller synthesis, they provide numerous advantages compared to centralized systems:

- the failure of one agent does not inevitably mean that the mission to be accomplished by the multi-agent system fails;
- a single agent can be easily replaced;
- the number of agents can be altered according to the specified task;
- less computational power and communication capabilities required.

In this paper we consider a formation of N identical quad-rotor helicopters that can communicate with each other to achieve the common goal. Further, each quad-rotor is controlled locally, as shown in Fig. 1. The controller has two components:

- K_L is a local feedback controller, used to internally stabilize the quad-rotor,
- K_F is a "formation-level" controller, receiving and compensating the error of the quad-rotor in the formation.

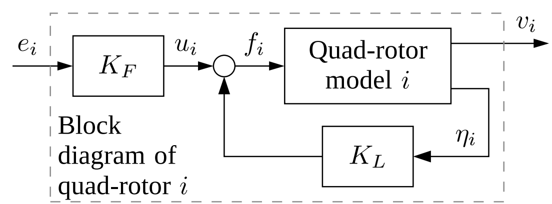

Fig. 1. Single quad-rotor and its controller

Here we use a 6-DOF linear model of the quad-rotor helicopters, and the graph-theory framework proposed in [6] to model the communication-topology between them. For a fixed number of LTI agents and known (and fixed) communication topology there are numerous controller design methods in the literature, see, e.g., [7], [8]. However, since in practice the number N of quad-rotors in the formation can vary with the specific task, and the communication topology might change during operation (agents getting far away, broken communication links, etc.), here we use a result on robust design proposed in [9]. This method guarantees that a formation controller K_F satisfying the design requirements, will stabilize any formation (with arbitrary number of agents and time-varying communication topology) with a given type of agent. We show how to extend the method to handle performance requirements, such as mixed-sensitivity constraints [10].

The rest of the paper is organized as follows. Section II briefly recalls the graph-theoretical framework proposed in [6] and the robust design method from [9], and shows how to incorporate performance requirements in the latter. The introduction of the quad-rotor system and the design of the local feedback controller K_L is carried out in Section III. The design of the formation-level controller K_F and simulation results are presented in Section IV. Finally, conclusions are drawn and an outlook to future work is given in Section V.

The following notation will be used throughout the paper: I_p denotes the $p \times p$ identity matrix; $\mathbf{R}^{p\times q}$, $\mathbf{C}^{p\times q}$ are, correspondingly, the sets of $p\times q$ real and complex matrices; diag $(x_1, \ldots, x_n)$ indicates an $n \times n$ diagonal matrix with the elements x_1 to x_n on the diagonal; $\otimes$ is the Kronecker product; $\mathrm{lft}(P(s), K(s))$ denotes the lower linear fractional transformation of $P(s)$ with $K(s)$ (see, e.g., [10]).

II. Formation Modeling and Controller Design

In this section we review the framework for formation control proposed in [6] and the controller design approach for stabilization of all formations proposed in [9]. Then we show how performance requirements can be incorporated.

A. Communication Topology and Graph-Theory

Consider a formation of N identical quad-rotors that are communicating with each other. As in [6], the communication topology is represented as a directed graph, where the nodes are the quad-rotors and the vertices are the communication links; an example is shown in Fig. 2.

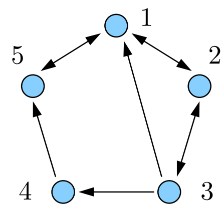

Fig. 2. Graph representation of a formation with $N = 5$ quad-rotors. In this example quad-rotor 3 receives information from quad-rotor 2 and sends information to quad-rotors 1, 2 and 4

In the following we assume no communication delays, or if any, they are fixed and accounted for in the quad-rotor dynamics.

Let $\mathcal{J}_i$ denote the set of quad-rotors (neighbors) from which quad-rotor i receives information, and $|\mathcal{J}_i|$ its size (the number of sensed neighbors). Then one measure of the relative error of quad-rotor i in the formation is the weighted sum of the errors towards the sensed neighbors

$$e_i = \frac{1}{|\mathcal{J}_i|} \sum_{k \in \mathcal{J}_i} e_{ik} \tag{1}$$

The term e_{ik} is the error between the i-th and k-th unit

$$e_{ik} = (r_i - v_i) - (r_k - v_k) = \bar{r}_{ik} - (v_i - v_k)$$

where r_i is a commanded absolute position for agent i, $\bar{r}_{ik}$ is a commanded (directional) offset between quad-rotor helicopters i and k which defines the desired formation given in relative coordinates, and v_i is the transmitted output of quad-rotor i. Here we assume that the outputs v_i are the coordinates of the quad-rotor i in the x, y and z coordinates (i.e., $v_i = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^T$), but this can be easily extended to include the quad-rotor orientations also.

In this paper we represent the communication topology using the normalized graph Laplacian matrix L which is defined as follows

$$L_{ik} = \begin{cases} 1, & \text{if } i = k \\ -\frac{1}{|\mathcal{J}_i|}, & k \in \mathcal{J}_i \\ 0, & k \notin \mathcal{J}_i \end{cases}$$

The important properties of L, which simplify the controller synthesis later on, are (see [6], [11])

- zero is an eigenvalue of L if the formation has no leader (i.e., there is no agent that does not receive information from any of the other agents),
- all eigenvalues λ_i of L lie in a unit disk centered at $1 + j0$ in the complex plane (the so called Perron disk),
- for undirected graphs L has only real eigenvalues.

Collecting reference, output and error vectors for the whole formation in a single column vector, (e.g. $\mathbf{v} = \begin{bmatrix} v_1^T & \dots & v_N^T \end{bmatrix}^T$), equation (1) can be rewritten as

$$\mathbf{e} = L_{(p)}(\mathbf{r} - \mathbf{v}) = \bar{\mathbf{r}} - L_{(p)}\mathbf{v} \tag{2}$$

where $L_{(p)} = L \otimes I_p$; here p is the number of outputs of v_i (in the above the number of selected outputs is $p = 3$). Note that since 0 is always an eigenvalue of L, $L_{(p)}$ is rank deficient and the same formation shape $\bar{\mathbf{r}}$ corresponds to multiple absolute coordinates $\mathbf{r}$.

The quad-rotor formation closed-loop system is shown in Fig. 3, where $H(s)$ includes everything in the dashed box in Fig. 1. Note that the only exchange of information between the quad-rotors occurs via the communication channels, i.e., through $L_{(p)}$.

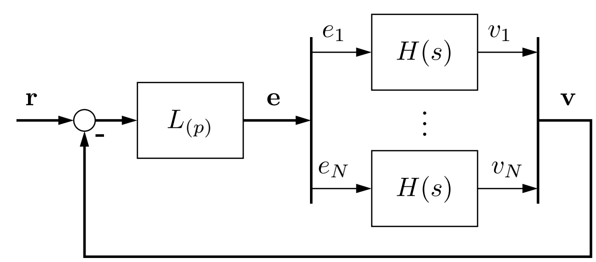

Fig. 3. Closed-loop representation of the formation

B. Closed-Loop Formation Stability

Definition 1 *A formation is called stable, if all poles of the closed-loop interconnection in Fig. 3 are strictly in the left half-plane.*

Let the stable dynamics of the closed-loop system formed by each quad-rotor and its local controller K_L be described by

$$\begin{aligned} \dot{\xi}_i &= A\xi_i + Bu_i \\ v_i &= C\xi_i \qquad i = 1, \dots, N \end{aligned} \tag{3}$$

where $A \in \mathbf{R}^{n \times n}$, $B \in \mathbf{R}^{n \times l}$ and $C \in \mathbf{R}^{p \times n}$.

Let the formation-level controller be an LTI controller $K_F(s)$, such that $u_i(s) = K_F(s)\, e_i(s)$. The following result is adapted from [6].

Theorem 1 *The controller $K_F(s)$ stabilizes the closed-loop formation (Fig. 3) if and only if it simultaneously stabilizes the set of N systems*

$$\begin{aligned} \dot{\tilde{\xi}}_i &= A\tilde{\xi}_i + B\tilde{u}_i \\ \tilde{v}_i &= \lambda_i C\tilde{\xi}_i \qquad i = 1, \dots, N \end{aligned} \tag{4}$$

where λ_i denotes an eigenvalue of L. The theorem can be proven by applying a state-, input- and output-transformation (Q) to the closed-loop formation, using the eigenvalue decomposition $L = Q\Lambda Q^{-1}$ (or the Schur decomposition

$L = QUQ^H$), thus resulting in a diagonal (block upper-triangular) system with the above N systems on the diagonal. Note that since the eigenvalues λ can be complex the above theorem requires the simultaneous stabilization of complex systems.

C. Robust Controller Design Approach

Several distributed design approaches to the formation control problem have been proposed in the literature. The methods presented in [8], [12] consider only state-feedback and are thus not suitable for application to quad-rotor formations because of the large communication overhead. A powerful design method for distributed controller design is proposed in [7], but since in the general case (directed graphs) it requires solving $2N$ output-feedback linear matrix inequalities (LMIs) together, the design can be very expensive for large formations or agents with many states (each LMI corresponds to a standard H_2 or H_∞ output feedback controller design formulation, see e.g. [10], [13]).

The approach we use here is proposed in [9]. Recalling that the eigenvalues λ (of every possible) normalized graph Laplacian L satisfy

$$\lambda = \{1 + \delta_\lambda \mid \delta_\lambda \in \mathbf{C},\ |\delta_\lambda| \leq 1\}$$

and using a standard approach from robust control [10], one can represent the group of N quad-rotors as a single one with uncertainty δ_λ and prove the following theorem.

Theorem 2 *A controller $K_F(s)$ stabilizes the closed-loop formation in Fig. 3 for any number of agents and any fixed communication topology if the system $G(s)$*

$$\begin{aligned} \dot{\tilde{\xi}} &= A\tilde{\xi} + B\tilde{u} \\ z_\delta &= C_\delta \tilde{\xi} \\ \tilde{v} &= C\tilde{\xi} + D_\delta w_\delta \\ w_\delta &= \delta_\lambda I_q z_\delta \end{aligned} \qquad (5)$$

is stable for all $|\delta_\lambda| \leq 1$, where $D_\delta C_\delta = C$.

The proof can be found in [9]. The factorization of C into $C_\delta \in \mathbf{R}^{q\times n}$ and $D_\delta \in \mathbf{R}^{p\times q}$ can be achieved using, e.g., singular value decomposition.

Consider now a norm bounded uncertainty $\Delta \in \boldsymbol{\Delta}$

$$\boldsymbol{\Delta} := \{\Delta \mid \Delta \in \mathbf{C}^{q\times q},\ \|\Delta\| \leq 1\}$$

acting on $G(s)$, $w = \Delta z$. The following theorem is adapted from [9].

Theorem 3 *A controller $K_F(s)$ stabilizes the closed-loop formation in Fig. 3 for any number of agents and any fixed as well as (arbitrary fast) time-varying communication topology if $\|T(s)\|_\infty < 1$, where $T(s) = \operatorname{lft}(G(s), -K(s))$ as shown in Fig. 4(a).*

The above theorem reduces the formation stability problem to an H_∞ design problem for a single quad-rotor with uncertainty, and guarantees that if a controller $K_F(s)$ satisfies the design requirements, any formation (with arbitrary communication topology) will be stable. However the theorem does not take into account the structure of the uncertainty (i.e., $\Delta = \delta_\lambda I_q$) and therefore leads to conservative results. By accounting for the structure of the uncertainty and using results from [14] one can prove the following theorem.

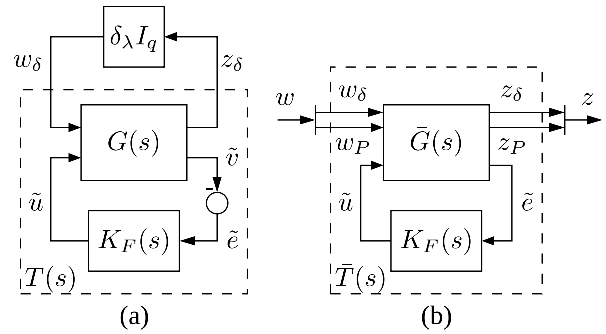

Fig. 4. LFT interconnection for robust stability design

Theorem 4 *A controller $K_F(s)$ stabilizes the closed-loop formation in Fig. 3 for any number of agents and any fixed as well as time-varying communication topology if there exists an invertible matrix $D \in \mathbf{R}^{q\times q}$ such that*

$$\|DT(s)D^{-1}\|_\infty < 1,$$

where $T(s) = \operatorname{lft}(G(s), -K(s))$ as shown in Fig. 4(a).

The above theorem results in a scaled H_∞ condition and can be used for controller synthesis using standard robust control tools.

D. Performance Requirements

Stability by itself is rarely sufficient for satisfactory control, and usually performance requirements have to be imposed. In fact, in order to obtain a meaningful formation-level controller, the performance requirements are a must, as shown by the next lemma.

Lemma 1 *Consider the closed-loop formation in Fig. 3 and a controller as shown in Fig. 1. There exists a formation-level controller K_F stabilizing the closed-loop formation if and only if the closed-loop of the system and the local-level controller K_L is stable. If, in addition, no performance requirements are imposed, $K_F(s) = 0$ is such a stabilizing controller.*

Proof: First assume that the local-feedback loop is unstable. Then, since 0 is always an eigenvalue of L one of the systems in (4) will be unstable and unobservable, hence no formation-level controller can stabilize it. Next, if the local-feedback loop is stable, so are the systems in (4) and one can select $K_F(s) = 0$, thus decoupling the systems while preserving stability. ∎

Next we show how mixed-sensitivity performance requirements [13] can be incorporated into the design. The generalized plant $\bar{G}(s)$ augmented with the exogenous inputs

w_P and performance outputs z_P is shown in Fig. 4(b). The construction of such a generalized plant with sensitivity $W_S(s)$ and control-sensitivity $W_K(s)$ filters is shown in Fig. 5, where $w_P = r$.

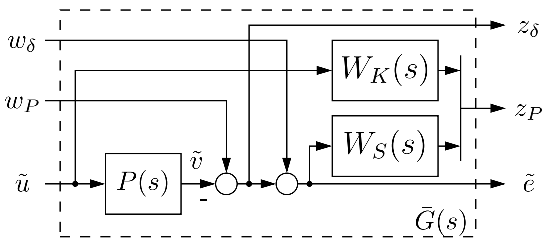

Fig. 5. Generalized plant with performance and uncertainty channels

Note that the sensitivity filter in the above generalized plant $\bar{G}(s)$ imposes a penalty on the relative error $\mathbf{e}$ of the quad-rotor's in the formation, i.e.,

$$\tilde{e} = \lambda(\tilde{r} - \tilde{v}) = \tilde{r} - \tilde{v} + \delta_\lambda(\tilde{r} - \tilde{v}).$$

If one wants to impose a requirement on the error between the quad-rotor position and the commanded (absolute) position $\mathbf{r} - \mathbf{v}$ instead, one needs to construct a corresponding generalized plant with $\tilde{r} - \tilde{v}$ as output. Because later we use a virtual-leader approach to give reference for the formation position we use the relative error.

Consider the closed-loop formation in Fig. 3, but augmented with shaping filters, and let $\mathbf{M}(s)$ be the corresponding transfer function. The design steps of transforming this closed-loop formation to the robust performance system (Fig. 5) can be summarized as:

1) Diagonalize the closed-loop formation $\tilde{\mathbf{M}}(s) = Q_p^{-1}\mathbf{M}(s)Q_p$, where $Q_{(p)} = Q \otimes I$.
2) Extract the systems $\tilde{M}_i(s)$ from the diagonal of $\tilde{\mathbf{M}}(s)$.
3) Combine the systems $\tilde{M}_i(s)$ to a single system with uncertainty $\tilde{M}(s)$.

Thus in order to relate the obtained performance index $\|\tilde{M}(s)\|_\infty$ to the performance of the original formation we need to look into the above transformation steps. Since $\tilde{M}(s)$ includes all $\tilde{M}_i(s)$ (included in the uncertainty representation) it follows that $\|\tilde{M}_i(s)\|_\infty \leq \|\tilde{M}(s)\|_\infty$. Now, if the eigenvector diagonalization is used in step 1, the system $\tilde{\mathbf{M}}(s)$ is diagonal and hence $\|\tilde{\mathbf{M}}(s)\|_\infty = \max_i \|\tilde{M}_i(s)\|_\infty \leq \|\tilde{M}(s)\|_\infty$. Finally, applying the Cauchy-Schwartz inequality on the transformation $\tilde{\mathbf{M}}(s) = Q_p^{-1}\mathbf{M}(s)Q_p$ leads to

$$\frac{\underline{\sigma}(Q)}{\bar{\sigma}(Q)}\|\tilde{M}(s)\|_\infty \;\leq\; \|\mathbf{M}(s)\|_\infty \;\leq\; \frac{\bar{\sigma}(Q)}{\underline{\sigma}(Q)}\|\tilde{M}(s)\|_\infty, \quad (6)$$

where $\underline{\sigma}(Q)$ and $\bar{\sigma}(Q)$ are correspondingly the smallest and the largest singular values of Q.

However, as pointed out in [7], the inequality becomes equality only if $L = L^T$, which is a (very) small subset of all possible topologies. Furthermore, even for very simple communication topologies the Laplacian is defective (e.g., the topology in Fig. 2, but with units 4 and 5 removed, has an eigenvalue with algebraic multiplicity 2 but geometric multiplicity 1) and hence $\underline{\sigma}(Q) = 0$. If, on the other hand, the Schur decomposition of L is used $\bar{\sigma}(Q) = \underline{\sigma}(Q) = 1$, but $\tilde{\mathbf{M}}(s)$ is upper triangular and no (simple) expression relates $\|\tilde{\mathbf{M}}(s)\|_\infty$ to $\|\tilde{M}_i(s)\|_\infty$. Nevertheless, although no analytic guarantees can be given for the formation performance under every topology, the robust performance design provides controllers stabilizing the formation and achieving good closed-loop behavior, as demonstrated in Section IV.

III. The quad-rotor helicopter

The aerial vehicle which serves as a basis for the analysis of the formation control algorithm is an electric quad-rotor aircraft which can be seen in Fig. 6 together with an earth-fixed frame E and a body-fixed frame B. We use Euler angles parametrization, and hence the airframe orientation in space is given by a rotation matrix $R \in SO3$ from B to E. We choose the quad-rotor helicopter as a platform for our formation control framework because it is considered in many research projects, [15], [16], [17]. The quad-rotor helicopter is a vertical takeoff and landing vehicle (VTOL) that has the ability to hover above a desired waypoint. Four motors are attached to the ends of a rigid frame shaped like a cross.

The front and rear rotors rotate counter-clockwise while the left and right rotors rotate clockwise as shown in Fig. 6. Ideally, gyroscopic effects and aerodynamic torques cancel out because of this configuration. The altitude of this aircraft is controlled by the combined throttle input which consists of the thrust of all four motors. A movement in direction of the x-axis is achieved by a change of the differential speed of motor M_1 and motor M_3, causing the quad-rotor helicopter to tilt around the y-axis. Similarly, a movement in direction of the y-axis can be obtained by a change of the differential speed of motor M_2 and motor M_4, which causes the quad-rotor helicopter to tilt around the x-axis. Finally, rotating the quad-rotor helicopter around the z-axis to cause a displacement in the yaw angle is achieved by increasing or decreasing the throttle to the motors M_1 and M_3. The throttle to the motors M_2 and M_4 has to be decreased or increased by the same amount to keep the total thrust and therefore the altitude constant.

A. Dynamical model

The dynamics of the quad-rotor aircraft are modeled by the following equations [15]

$$m\ddot{x} = -u\sin(\theta) \quad (7)$$

$$m\ddot{y} = u\cos(\theta)\sin(\phi) \quad (8)$$

$$m\ddot{z} = u\cos(\theta)\cos(\phi) - mg \quad (9)$$

$$\ddot{\psi} = \tau_\psi, \qquad \ddot{\theta} = \tau_\theta, \qquad \ddot{\phi} = \tau_\phi \quad (10)$$

where x and y describe the position of the quad-rotor helicopter in the horizontal plane, and z is the vertical coordinate; ϕ is the roll angle around the x-axis, θ is the pitch angle around the y-axis and ψ is the yaw angle around the z-axis; u is the thrust directed out of the center of gravity of

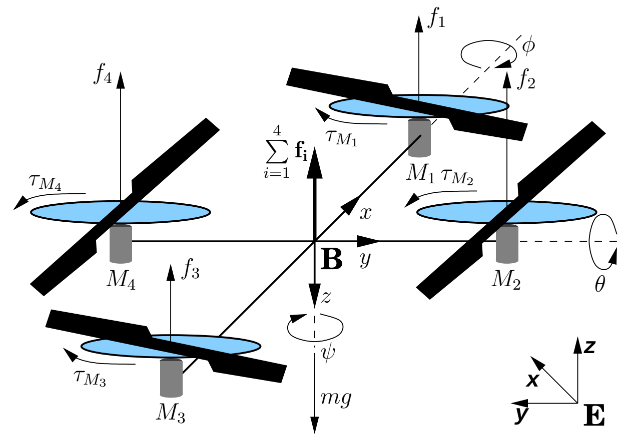

Fig. 6. Representation of the coordinate system of the quad-rotor helicopter

the aircraft and τ_ϕ, τ_θ and τ_ψ are the torques; $m = 0.64$ kg is the total mass of the quad-rotor helicopter and $g = 9.81\,\text{m/s}^2$ describes the gravitational constant.

Based on the approach in [15], we use the linearized model with the following state equations

$$\xi = \begin{bmatrix} x & \dot{x} & y & \dot{y} & z & \dot{z} & \psi & \dot{\psi} & \theta & \dot{\theta} & \phi & \dot{\phi} \end{bmatrix}^T.$$

The control inputs are defined as follows

$$\begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T = \begin{bmatrix} u - mg & \tau_\psi & \tau_\theta & \tau_\phi \end{bmatrix}^T.$$

The state equation can be written as $\dot{\xi} = f(\xi, u)$. Now assume that $\xi = 0$ is an equilibrium point with $f(0,0) = 0$. The first order Taylor expansion at the origin yields the linear system

$$\dot{\xi} = A\xi + B\bar{u} \tag{11}$$

where

$$\bar{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T.$$

B. Control strategy and simulation results

Using the linearized model of Section III-A, one can design a linear controller to stabilize a single quad-rotor helicopter. We assume that the sensors of the quad-rotor are capable of measuring all the states of the model. Therefore we choose a full state feedback LQR controller $u = -K_L\xi$. The choices of the weighting matrices Q and R, as well as the state feedback controller K_L can be found in Appendix A.

To verify closed-loop stability of a single quad-rotor helicopter we perform a simulation with the initial conditions

$$(x_i, y_i, z_i, \psi_i, \theta_i, \phi_i) \;= (-10\,\text{m}, 10\,\text{m}, 0\,\text{m}, 50°, 20°, 20°)\,.$$

The new setpoint is

$$(x_d, y_d, z_d, \psi_d, \theta_d, \phi_d) = (0\,\text{m}, 0\,\text{m}, 10\,\text{m}, 0°, 0°, 0°)\,.$$

Figs. 7 and 8 show the results of the simulation.

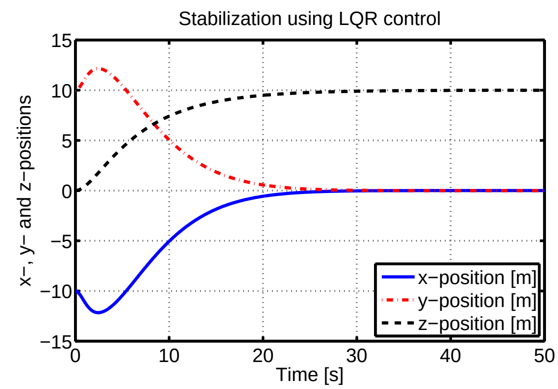

Fig. 7. Position of the quad-rotor helicopter

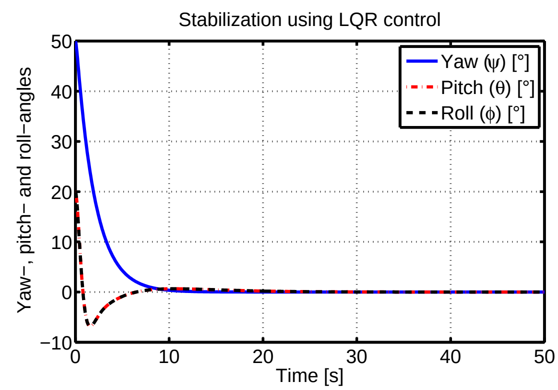

Fig. 8. Orientation of the quad-rotor helicopter

IV. Simulation results of a formation flight

This section shows an application of the proposed design method in Section II. For the linear model of a quad-rotor given in Section III-A and the stabilizing controller K_L obtained above we design a controller K_F using a S/KS mixed sensitivity approach in order to meet performance requirements.

The output signal of the formation contains translational position information of the quad-rotor helicopters (x-, y- and z-positions), weighted by sensitivity shaping filters. The filter for control sensitivity uses all four inputs of the quad-rotor helicopter. Both filters are given in Appendix B. The $\mathcal{H}_\infty$-norm between w_δ and z_δ (see Fig. 4.a) smaller than 1 means that the formation is robust against sudden changes in the communication topology and moreover, the formation reaches the desired waypoint in a specified time with a small steady-state error.

In our simulation the communication topology suddenly changes three times each at 10, 20 and 30 s. In this simulation we use a target waypoint with no dynamics to force the quad-rotors to approach an absolute position. Due to space constraints only the results for the x-axis are given in Fig. 9. The simulation shows that after 15 s the whole group of agents reaches the relative positions of the prespecified formation although there has been a change in the communication topology already. In addition, the formation is able to handle multiple changes in the communication topology, and continues to approach the waypoint. The comparison between the simulation results for the single quad-rotor and the formation response illustrates the price in terms of speed of response that has to be paid for guaranteeing stability for all possible communication topologies. A movie showing the simulated flight of the formation is available at the website: *http://www.tu-harburg.de/rts – "Material without password" – "Formation control"*.

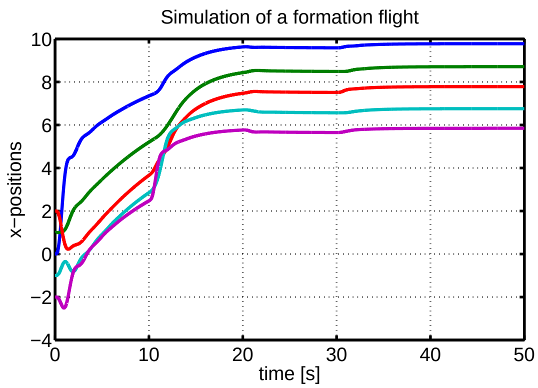

Fig. 9. Simulated x-positions of the formation

V. Conclusions and Future Work

In this paper the design of local controllers for a formation of identical vehicles is investigated in a case study - here a formation of dynamically unstable and under-actuated quad-rotor helicopters is considered.

The quad-rotors themselves are stabilized by local controllers. In addition, local formation controllers - designed as robust controllers for a single vehicle with uncertainty - guarantee stability of the whole formation of vehicles for any time-varying communication topology and any number N of units. Performance requirements are incorporated into the design using mixed-sensitivity loop shaping. It can also be shown that the formation is robust to arbitrary switches in the communication topology.

Nevertheless the robustness against all time-varying communication topologies provided by the distributed controller synthesis approach used here may result in an inferior system performance if the communication topology is known, or if the class of admissible topologies is restricted. Future research will aim at reducing this conservatism by designing controllers that incorporate *a priori* knowledge about topologies that are not admitted.

Appendix

A. LQR controller

The weighting matrices of the LQR controller are
$R = \text{diag}\,(100, 0.1, 25, 25)$ and
$Q = \text{diag}\left(0.04, 1, 0.04, 1, 0.5, 20, 0.25, 1, 10^3, 50, 10^3, 50\right).$

The controller in Section III-B has the following form

$$K_L = \begin{bmatrix} K_{L1} & K_{L2} \end{bmatrix}, \qquad K_{L1} = \begin{bmatrix} 0_{2\times 4} \\ I_2 \otimes \begin{bmatrix} -0.04 & -0.33 \end{bmatrix} \end{bmatrix}$$
$$K_{L2} = \text{diag}(\begin{bmatrix} 0.07 & 0.54 \end{bmatrix}, \begin{bmatrix} 5.00 & 10.49 \end{bmatrix}, I_2 \otimes \begin{bmatrix} 8.16 & 4.28 \end{bmatrix}).$$

B. Mixed-sensitivity design

The sensitivity and control sensitivity weighting filters are

$$W_S = I_3 \otimes \left(\frac{3.333}{s+0.01}\right), \quad W_K = I_4 \otimes \left(10^2 \frac{s+10^3}{s+10^6}\right).$$

The designed $\mathcal{H}_\infty$ controller is of 19^{th} order. The robust performance and robust stability H_∞ norms are correspondingly

$$\|D\bar{T}(s)D^{-1}\|_\infty = 3.226 \text{ and } \|\bar{T}_{z_\delta w_\delta}(s)\|_\infty = 0.9946 < 1.$$

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